# Jordan-von Neumann Characterization of Inner Products

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- In 1932 , S. Banach ( "Theorie des Operations Lineaires" , Monografje Matematyczne, Warsaw,1932 , Poland ) introduced the normed linear spaces. In 1935, P. Jordan & J. von Neumann ( Ann. Math. 36 (1935) , 719-723 ) established the following parallelogram equality characterization of inner product spaces .

*Jordan - von Neumann Characterization of Inner Products* : A normed linear space $V$ is an inner product space if the parallelogram equality :
$\|v-w\|^2+\|v+w\|^2=2\|v\|^2+2\|w\|^2 \quad (v\,,\,w\in V)$ holds .

A Banach space whose norm satisfies this law is a Hilbert space . G. Birkhoff ( Duke Math. J. , 1 (1935) , 169-172 ) , R. C. James ( Duke Math. J. , 12 (1945) , 291-301 ; Bull. Amer. Math. Soc. 53 (1947) , 559-566 ) and M. M. Day ( Trans. Amer. Math. Soc. , 62 (1947) , 320-337 ) gave basic characterizations of inner products by orthogonality relations and by the duality map .Moreover, a major improvement was the idea of I. J. Schoenberg ( Proc. Amer. Math. Soc. , 3 (1952) , 961-964 ) to replace the parallelogram equality by an inequality. The most "natural" geometric properties may fail to hold in a general normed space unless the space is an inner ( or : scalar ) product space . These characterizations of inner products or Hilbert spaces are important in the geometry of Banach spaces and approximation theory .