# CURRICULUM VITAE (C.V.)

# John Michael Rassias

# Education

- Elementary School : Pellana (Sparta), Greece
- High School : Castorion (Sparta), Greece
- Undergraduate: Aristotelion University of Thessaloniki, Mathematics Department, Greece
- Military Education and Service : Greece
- Graduate :
  - **George Washington University**, Mathematics Department, 1972-1973
  - o Stanford University, Mathematics Department, 1973-1975 : M. S.
  - University of California, Berkeley, Mathematics Department, 1975-1977 : Ph. D.
     Dissortation: Mixed Type Partial Differential Equations in P<sup>n</sup>

Dissertation: *Mixed Type Partial Differential Equations in*  $\mathbb{R}^n$ . Advisor : **Murray Harold PROTTER**. Members: **SH. KOBAYASHI** and **R. K. SACHS** 

# **Positions Held**

- University of California , Berkeley , USA
   Research Assistant of Mathematics, 1975-1977
  - National and Technical University of Athens, Greece
  - Research Fellow of Mathematics , 1978
  - American College of Greece, Athens, Greece
    - Assistant Professor of Mathematics , 1981-1985
    - Associate Professor, 1985-1992
- University of Maryland , Campus in Athens , Greece
   Adjunct Professor of Mathematics , 1982-1983
- Embry-Riddle University, Campus in Athens, Greece
  - Adjunct Professor of Mathematics , 1983-1984
- Hellenic Military Academy , Greece
- Adjunct Professor of Mathematics, 1982-1983; 1985-1986
- Hellenic Airforce Academy, Greece
  - Adjunct Professor of Mathematics & Theoretical Mechanics, 1984 ; 1988
- National and Capodistrian University of Athens , Greece
  - o Lecturer, 1989-1992
  - o Assistant Professor, 1992-1996
  - o Associate Professor, 1996-2000
  - o Professor of Mathematics, 2000-

# **Journal Editor**

- Journal of Inequalities in Pure and Applied Mathematics (JIPAM)
- Australian Journal of Mathematical Analysis and Applications (AJMAA)
- Journal of Applied Mathematics and Stochastic Analysis (JAMSA)
- Abstract and Applied Analysis (AAA)
- Journal of Mathematics and Statistics (JMS)
- International Journal of Applied Mathematics and Statistics (IJAMAS)
- Mathematical Sciences Research Journal (MSRJ)
- Global Journal of Pure and Applied Mathematics (GJPAM)
- International Journal of Pure and Applied Mathematical Sciences (IJPAMS)
- International Journal of Theoretical and Applied Mathematics (IJTAM)
- Pacific-Asian Journal of Mathematics (PAJM)
- Global Journal of Applied Mathematics and Mathematical Sciences (GJAMMS)
- Journal of Analysis and Computation (JAC)
- International Journal of Evolution Equations (IJEE)
- International Journal of Mathematics and Systems (IJMS)
- African Diaspora Journal of Mathematics (ADJM)
- Communications in Mathematical Analysis (CMA)
- Journal of Applied Mathematical Analysis (JAMA)
- International Journal of Mathematics and Analysis (IJMA)
- PanAmerican Mathematical Journal (PMJ)
- Far East Journal of Mathematical Sciences (FJMS)
- Advances in Theoretical and Applied Mathematics (ATAM)
- Applied Sciences-Electronic Journal (ASEJ)
- The Oriental Journal of Mathematical Sciences (OJMS)
- The International Journal of Contemporary Mathematical Sciences (IJCMS), Bulgaria
- The Mathematical Atlas (MA), USA
- International Journal of Applied Mathematics and Engineering Sciences (IJAMES)
- Journal of Approximation Theory and Applications (JATA)
- Journal of Applicable Functional Differential Equations (JAFDE)
- Journal of Mathematical Analysis and Approximation Theory (JMAAT)
- International Journal of Approximation Theory and Applications (IJATA)
- Oriental Journal of Pure and Applied Mathematics (OJPAM)
- Journal of Nonlinear Functional Analysis and Differential Equations (JNFADE)
- Asian-African Journal of Mathematics and Mathematical Sciences (AAJMMS)
- International Journal of Nonlinear Operators Theory and Applications (IJNOTA)
- Indian Journal of Mathematics and Mathematical Sciences (IJMMS)
- Journal of Inequalities and Approximation Theory (JIAT)
- Arabian Journal of Mathematics and Mathematical Sciences (AJMMS)
- International Journal of Nonlinear Dynamical Systems and Chaos (IJNDSC)
- International Journal of Computing and Mathematical Applications (IJCMA)

- Communications in Differential and Difference Equations (CDDE)
- WSEAS Transactions on Mathematics (WSEAS-TM))
- Journal of Concrete and Applicable Mathematics (JCAAM)

### Announcements

- University of Sao Paulo, Brazil
  - Special Visiting Professor of Mathematics, 1988
  - University of Turin , Italy
    - o Visiting Professor of Mathematics, 1989

## **Areas of Specialization**

- <u>Mixed Type Partial Differential Equations</u> (MR 35M)
- <u>Functional Equations and Inequalities</u> (MR 39B)
- Operator Theory (MR 42, 46C, 47D)
- <u>Mathematical Inequalities</u> (MR 26D, 30C, 60, 62)

### **Selected Addresses**

- Functional Differential Systems and Related Topics, Institute of Mathematics, Polish Academy of Sciences, 1979, Poland
- Summer Mathematical School, U.N.E.S.C.O., 1980, Greece
- *On Singularities in Boundary Value Problems*, N.A.T.O., Advanced Study Institute, Maratea, 1980, Italy
- Symposium on Numerical Analysis and Computational Complex Analysis, E.T.H., 1983, Switzerland
- American Mathematical Society Meeting, 1987, USA
- 94<sup>th</sup> Annual Meeting of the American Mathematical Society, 1988, USA
- Technical Institute, Mathematics Department, Zielona Gora, 1990, Poland
- University of Sofia, Mathematics Department, 1991, Bulgaria
- 1018<sup>*H*</sup> AMS Meeting, San Francisco, California, April 29-30, 2006, USA.

## Memberships

- Hellenic Mathematical Society
- American Mathematical Society
- California Alumni Association
- American Association for the Advancement of Science
- Sigma Xi, The Scientific Research Society
- The New York Academy of Sciences
- Academy of Arts and Sciences of the Americas
- International Academy of Creative Endeavors (IACE), Russia
- Royal Astronomical Society, London, UK
- The Marquis Who's Who Publ. Board in Sciences and Engineering
- International Society of Difference Equations (ISDE)
- Research Group in Mathematical Inequalities and Applications

### (RGMIA), Victoria University, Australia

### **Reviewer - Referee**

- Mathematical Reviews , American Mathematical Society , USA
- Hellenic Mathematical Society, Greece

# **Referee/Reviewer for the following 8 journals**

- Journal of Mathematical Analysis and Applications (JMAA)
- Applied Mathematics E-Notes (AMEN)
- Applied Mathematics Letters (AML)
- Journal of Difference Equations and Applications (JDEA)
- International Journal of Mathematics and Mathematical Sciences (IJMMS)
- Acta Mathematica Sinica (AMS)
- Science in China: Mathematics (SCM)
- Taiwanese Journal of Mathematics (TJM)

# **Referee/Reviewer for Springer/Birkhauser Scientific Books**

# **Books Authored**

- Vector Calculus and Analytic Geometry, 1980, Evos Publ., Greece
- *Mixed type Equations and Maximum Principles in fluid dynamics*, 1983, Greece
- Probability Theory and Statistics, 1989, Symmetry Publ., Greece
- Number Theory, 1991, Symmetry Publ., Greece
- Linear Algebra and Linear Programming, 1993, Symmetry Publ., Greece
- *Mathematical Analysis : Differential Calculus*, Vol. I, 1994, Symmetry Publ., Greece
- *Mathematical Analysis : Integral Calculus*, Vol. II, 1995, Symmetry Publ., Greece
- *Mathematical Analysis : Multivariable Calculus*, Vol. III, 1996, Symmetry Publ., Greece
- Applied Analysis : Differential Geometry , Vol. I , 2002 , Symmetry Publ. , Greece
- Applied Analysis : Partial Differential Equations, Vol. II, 2003, Symmetry Publ., Greece
- Lecture Notes on Mixed Type Partial Differential Equations, World Scientific, 144 pp,1990
- Counter Examples in Differential Equations and Related Topics, World Scientific, 192 pp,1991

#### **Selected Citations**

- W. F. Ames (GIT) for the Mathematical Reviews, AMS: Any analyst is always concerned about good counterexamples to use in lectures and research. Here is a book with a collection of counterexamples in the areas of continuity, differentiability, extrema, existence, uniqueness, stability, regularity, periodicity, etc. They have been collected from a variety of books and journals. The collection should prove useful to analysts interested in the main topic of differential equations. There is a reasonable bibliography and both a subject and an author index. This is a valuable book for students and researchers alike.
- -G.C.Rota (M.I.T.) for the World Scientific : The author knows his business : he has chosen the right counterexamples at the right time. The section on plane autonomous systems is particularly felicitous, and should be consulted by whoever teaches elementary differential equations ( some of the examples are really cute and we will use them in our 400-student course next year ). Why can't mathematicians write more books like this one ?

### **Books Edited**

- *Mathematics Space Technology*, Greece, 1980
- Mathematical Analysis, Teubner-Texte zur Mathematik, Vol. 79, 1985
- Mixed type Equations, Teubner-Texte zur Mathematik, Vol. 90, 1986
- Functional Analysis, Approximation Theory and Numerical Analysis, World Scientific, 340 pp,1994
- Geometry, Analysis and Mechanics, World Scientific, 388 pp, 1995
- Advances in Equations and Inequalities, Hadronic Press, USA, 1999
- Mathematical Equations and Inequalities, Vol. I, Greece, 1999
- Mathematical Equations and Inequalities, Vol. II, Greece, 1999
- Functional Equations, Integral Equations and Differential Equations and Applications, Inter. J. Appl. Math. & Stat., 980 pp., 2007.

## **Selected Published Papers**

Well-posedness of the Tricomi boundary value problem for partial differential equations of mixed type, maximum principle of the Cauchy problem for hyperbolic differential equations, stability and asymptotic behavior of the Ulam problem for functional equations and inequalities, Jordan - von Neumann type characterizations of inner products, Landau extremum problem and Heisenberg uncertainty inequality.

- 1. *A counter example to a conjecture by P. Erdos*, (with G. Rassias, Th. Rassias) Proc. Japan Acad. Ser. A Math. Sci. **53** (1977), no. 4, 119-121
- 2. Weak solutions of the Frankl Morawetz problem in  $\mathbb{R}^{n+1}$   $(n \ge 2)$ , (with G. Rassias, Th. Rassias), Tamkang J. Math. **10** (1979), no. 1, 81-91

- 3. On the Frankl problem of second kind, Tamkang J. Math. **10** (1979), no. 2, 231-236
- 4. *The 3-dimensional Frankl problem*, Bull. Soc. Roy. Sci. Liege **48** (1979), no. 11-12, 422-423
- 5. *The Bitsadze-Lavrentjev problem*, Bull. Soc. Roy. Sci. Liege **48** (1979), no. 11-12, 424-425
- 6. *A new mixed type boundary value problem*, Bull. Soc. Roy. Sci. Liege **48** (1979), no. 11-12, 420-421
- Some fixed point theorems in nonlinear analysis, (with G. Rassias, Th. Rassias) "Functional differential systems and related topics" (Proc. First Internat. Conf., B \ I a\.zejewko,1979), 302-305, Higher College Engrg., Zielona Gora (1980)
- 8. *The survey on equations of mixed type*, "Functional differential systems and related topics" (Proc. First Internat. Conf., B \ I a\.zejewko,1979), 295-301, Higher College Engrg., Zielona Gora, (1980) 445-446.
- 9. *A maximum principle in R*<sup>3</sup>, C. R. Math. Rep. Acad. Sci. Canada **2** (1980), no.3, 131-133
- Research Problems in Complex Analysis (New Problem : No. 6.81), (with G. Rassias, Th. Rassias), (Eds.: D. A. Bannan and J. G. Clunie, Aspects of Contemp. Complex Analysis), Acad. Press, (1980), 584
- 11. New uniqueness theorems, Bull. Acad. Polon. Sci. Ser. Sci. Math. 28 (1980), no. 11-12, 569-571
- 12. *A new bi-hyperbolic boundary value problem in the Euclidean space*, Bull. Acad. Polon. Sci. Ser. Sci. Math. **28** (1980), no. 11-12, 565-568
- 13. On a defective theorem on elliptic hyperbolic equations, Bull. Soc. Roy. Sci. Liege **49** (1980), no. 9-10, 307-309
- 14. Weak solutions for a mixed type problem, Bull. Soc. Roy. Sci. Liege **49** (1980), no. 5-8, 278-280
- 15. *On a Goursat type problem*, C. R. Math. Rep. Acad. Sci. Canada **2** (1980/81), no. 1, 49-51
- Uniqueness and existence theorems for a mixed type equation, Tamkang J. Math. 12, (1981), no. 1, 77-83
- 17. *A new mixed type boundary value problem*, Bull. Sci. Math. (2) **105** (1981), no. 3, 329-336
- Mixed type partial differential equations in R<sup>n</sup>, Tamkang J. Math. 12 (1981), no. 2, 177-181
- A uniqueness theorem for the generalized Frankl-Tricomi problem, Bull. Sci. Math. (2) 105 (1981), no. 3, 321-327
- 20. A maximum principle in R<sup>n+1</sup>, J. Math. Anal. Appl. **85** (1982), no. 1, 106-113
- Weak solutions of the Frankl problem in the 4- dimensional Euclidean space, Bull. Acad. Polon. Sci. Ser. Sci. Math. 30 (1982), no. 3-4, 123-130
- 22. On the derivative of a polynomial, Bull. Soc. Roy. Sci. Liege **51** (1982), no. 9-10, 379-380
- 23. An application of the theory of positive symmetric systems to a degenerate multidimensional hyperbolic equation in R<sup>3</sup>, Serdica 8 (1982), no. 3, 235-242
- 24. On approximation of approximately linear mappings by linear mappings, J. Funct. Anal. **46** (1982), no. 1, 126-130
- 25. An extended Chaplygin problem and a uniqueness theorem for the Chaplygin Frankl problem, Bull. Soc. Roy. Sci. Liege **51** (1982), no. 3-4, 156-160

- 26. *The Bi-hyperbolic Degenerate Boundary Value Problem in R*<sup>3</sup>, Discuss. Math., Vol. 5 , (1982), 101-104
- 27. *The extended Bitsadze-Lavrentjev-Tricomi boundary value problem*, Rend. Circ. Mat. Palermo (2) **33** (1984), no.2, 255-264
- 28. On approximation of approximately linear mappings by linear mappings, Bull. Sci. Math. (2) **108** (1984), no. 4, 445-446
- 29. On the Tricomi problem with two parabolic lines of degeneracy, Bull. Inst. Math. Acad. Sinica **12** (1984), no. 1, 51-56
- 30. On the generalized Cesaro operators, Mathematical Analysis, 32-34, Teubner-Texte Math., 79, Teubner, Leipzig, (1985)
- 31. On a new approximation of approximately linear mappings by linear mappings, Discuss. Math. 7 (1985), 193-196
- 32. On the exterior mixed type boundary value problem in the Euclidean plane, "Mathematical Analysis", 269-284, Teubner-Texte Math., 79, Teubner, Leipzig, (1985)
- Extended Bitsadze-Lavrentjev problem with two parabolic lines of degeneracy and two elliptic arcs in Euclidean plane, C. R. Acad. Bulgare Sci. 38 (1985), no. 1, 31-34
- 34. *The mixed Bitsadze-Lavrentjev-Tricomi boundary value problem*, "Mixed Type Equations", 6-21, Teubner-Texte Math., 90, Teubner, Leipzig, (1986)
- 35. On three new uniqueness theorems of the Tricomi probem for nonlinear mixed type equations, "Mixed Type Equations", 269-279, Teubner-Texte Math., 90, Teubner, Leipzig, (1986)
- 36. *The mixed Bitsadze-Lavrentjev-Tricomi boundary value problem*, J. Math. Res. Expositions **7** (1987) ,no.1 , 77-80
- 37. Two new criteria on characterizations of inner products, Discuss. Math.
  9 (1988), 255-267
- On three new generalized uniqueness theorems of the Tricomi problem for nonlinear mixed type equations, J. Math. Phys. Sci. 22 (1988), no. 6, 681-695
- 39. On the well-posedness of the extended Chaplygin problem in a multidimensional region, C. R. Acad. Bulgare Sci. **41** (1988), no. 2, 35-37
- 40. Solution of a problem of Ulam, J. Approx. Theory 57 (1989), no. 3, 268-273
- 41. *The well-posed Tricomi-Bitsadze-Lavrentjev problem in the Euclidean plane*, Atti Accad. Sci. Torino Cl. Sci. Fis. Mat. Natur. **124** (1990), no. 3-4, 73-83
- 42. Four new criteria on characterizations of inner products, Discuss. Math. 10 (1990), 139-146
- 43. *The exterior Tricomi and Frankl problem*, J. Math. Res. Exposition **10** (1990) no. 4 , 485-493
- 44. On the well-posedness of the extended Tricomi-Chaplygin-Frankl problem in a multidimenional region, Chinese. J. Math. **19** (1991), no.3, 187-203
- 45. Solution of a stability problem of Ulam, Discuss. Math. 12 (1992), 95-103
- 46. On the well-posed Tricomi problem in  $\mathbb{R}^2$ , Discuss. Math. **12** (1992), 85-93
- 47. On the stability of the Euler-Lagrange functional equation, Chinese J. Math.
  20 (1992), no. 2, 185-190
- 48. *The well-posed Tricomi problem of two kinds*, J. Math. Phys. Sci. **27** (1993), no. 6, 383-393
- 49. Landau's type inequalities, C. R. Acad. Bulgare. Sci. 46 (1993), no.10, 9-11
- 50. *Open problems in analysis*, "Geometry, analysis and mechanics", 355-364, World Scientific Publishing Co., Inc., River Edge, NJ, (1994)

- 51. Stefan Banach, Alexander Markowic Ostrowski, Stanislaw Marcin Ulam, "Functional analysis, approximation theory and numerical analysis", 241-249, World Sci. Publishing, River Edge, NJ, (1994)
- 52. Complete solution of the multi-dimensional problem of Ulam, Discuss. Math. 14 (1994), 101-107
- 53. New Landau's type inequalities, Discuss. Math. 14 (1994), 77-99
- 54. *Archimedes*, "Geometry, Analysis and Mechanics", 1-4, World Sci. Publishing, River Edge, NJ, (1994)
- 55. On the stability of the multi-dimensional nonlinear Euler-Lagrange functional equation, "Geometry, Analysis and Mechanics", 275-285, World Sci. Publishing, River Edge, NJ, (1994)
- 56. On the stability of the non-linear Euler-Lagrange functional equation in real normed linear spaces, J. Math. Phys. Sci. **28** (1994), no.5, 231-235
- 57. On the stability of a multi-dimensional Cauchy type functional equation, "Geometry, Analysis and Mechanics", 365-376, World Sci. Publishing, River Edge, NJ, (1994)
- 58. *Fractional linear algebra*, "Geometry, Analysis and Mechanics", 251-267, World Sci. Publishing, River Edge, NJ, (1994)
- 59. Solution of a stability problem of Ulam, "Functional analysis, approximation theory and numerical analysis", 241-249, World Sci. Publishing, River Edge, NJ, (1994)
- 60. *Multi-dimensional Landau inequalities*, "Geometry, Analysis and Mechanics", 287-354, World Sci. Publishing, River Edge, NJ, (1994)
- 61. *The well-posed Tricomi problem in the Euclidean plane*, "Geometry, Analysis and Mechanics", 189-195, World Sci. Publishing, River Edge, NJ, (1994)
- 62. *A new computation formula for the inverse of a matrix*, "Geometry, Analysis and Mechanics", 197-203, World Sci. Publishing, River Edge, NJ, (1994)
- 63. *Landau's type inequalities*, "Functional analysis, approximation theory and numerical analysis", 281-301, World Sci. Publishing, River Edge, NJ, (1994)
- 64. *Generalized Landau's type inequalities*, "Functional analysis, approximation theory and numerical analysis", 303-325, World Sci. Publishing, River Edge, NJ, (1994)
- 65. On the extended Ostrowski constant, "Functional analysis, approximation theory and numerical analysis", 237-239, World Sci. Publishing, River Edge, NJ, (1994)
- 66. Landau's type inequalities, J. Math. Anal. Appl. **202** (1996), no. 1, 280-301
- 67. On the stability of the general Euler Lagrange functional equation, Demonstratio Math. **29** (1996), no. 4, 755-766
- 68. Uniqueness of quasi-regular solutions for a parabolic elliptic-hyperbolic Tricomi problem, Bull. Inst. Math. Acad. Sinica **25** (1997), no.4, 277-287
- 69. *Bitsadze-Lavrentjev Problem*, Encyclopaedia of Mathematics, KluwerAcademic Publishers, file: B: rassi 1, March 28, (1997), 1-4, The Netherlands
- Multi-dimensional Landau extremum problems, C. R. Acad. Bulgare Sci. 50 (1997), no. 2, 5-8
- 71. Solution of the Ulam stability problem for Euler-Lagrange quadratic mappings, J. Math. Anal. Appl. **220** (1998), no. 2, 613-639
- 72. Solution of the Ulam stability problem for quartic mappings, Glasnik Matematicki Ser. III **34 (54)** (1999), no. 2, 243-252

- 73. Existence of weak solutions for a parabolic elliptic-hyperbolic Tricomi problem, Tsukuba J. Math. 23 (1999), no. 1, 37-54
- 74. On the stability of the multi-dimensional Euler-Lagrange functional equation, J. Indian Math. Soc. (N. S.) **66** (1999), no. 1-4, 1-9
- 75. *Multi-dimensional Landau extremum problems*, J. Indian Math. Soc. (N. S.) 66 (1999), no. 1-4, 11-16
- 76. Six-dimensional Landau inequalities, Demonstratio Math. **32** (1999), no.2, 413-431
- 77. Solution of the General Dth Degree Functional equation, (A.E.I. : Advances in Equations and Inequalities), Hadronic Press, Fl., USA, (1999), 185-189
- Generalization of the Euler Theorem to Heptagons leading to a Quadratic Vector Identity, (A.E.I. : Advances in Equations and Inequalities), Hadronic Press, Fl., USA, (1999), 179-183
- 79. *Mathematical Computation of the Code of a Date*, (A.E.I. : Advances in Equations and Inequalities), Hadronic Press, Fl., USA, (1999), 191-210
- 80. *Augustin Louis Cauchy*, *His Life and His Work*, (A.E.I. : Advances in Equations and Inequalities), Hadronic Press, Fl., USA, (1999), 1-25
- 81. On the Euler stability problem, J. Indian Math. Soc. (N.S.) **67** (2000), no.1-4, 1-15
- Solution of the Ulam stability problem for nonlinear five-dimensional Euler quadratic mappings, Southeast Asian Bull. Math. (Springer-Verlag) 24 (2000), no. 4, 617-621
- 83. On approximation of approximately qudratic mappings by quadratic mappings, Ann. Math. Sil., No. 15 (2001), 67-78
- 84. Solution of a quadratic stability Hyers-Ulam type problem, Ricerche Mat. 50 (2001), no. 1, 9-17
- 85. Solution of a Cauchy Jensen stability Ulam type problem, Archivum Mathematicum (Brno) **37** (2001), no.3, 161-177(.pdf file)
- 86. Solution of the Ulam stability problem for cubic mappings, Glasnik Matematicki Ser. III **36** (**56**) (2001), no.1, 63-72
- 87. Hyers–Ulam stability for a quadratic functional equation in several variables, J. Indian Math. Soc. (N. S.) 68 (2001), no.1-4, 65 - 73
- 88. On the Ulam stability of mixed type mappings on restricted domains,
  J. Math. Anal. Appl. 276 (2002), no. 2, 747-762
- 89. On the Hyers-Ulam stability problem for quadratic multi-dimensional mappings, Aequationes Math. (Birkhauser-Verlag) 64 (2002), no.1-2, 62-69
- Uniqueness of quasi-regular solutions for a bi-parabolic elliptic bi-hyperbolic Tricomi problem, Complex Var. Theory Appl.(Taylor & Francis) 47 (2002), no. 8, 707-718
- 91. Solution of the Ulam stability problem for an Euler type quadratic functional equation, Southeast Asian Bull. Math. **26** (2002), 101-112
- On the Hyers-Ulam stability problem for quadratic multi-dimensional mappings on the Gaussian plane, Southeast Asian Bull. Math. 26 (2002), 483-502
- 93. On some approximately quadratic mappings being exactly quadratic, (with M. J. Rassias), J. Ind. Math. Soc. **69** (2002), 155-160.
- 94. On the Ulam stability of Jensen and Jensen type mappings on restricted domains, (with M. J. Rassias), J. Math. Anal. Appl. 281 (2003),516-524
- 95. On the quadratic functional inequality involving a sum of powers of norms, Intern. J. Math. Sciences 2 (2003), no. 1, 173-184

- 96. On the general quadratic functional equation, Libertas Math. ,23(2003), 165- 174.
- 97. On the Heisenberg Pauli Weyl inequality, J. Inequ. Pure & Appl. Math., 5(2004), Issue 1,1-70 pp.
- 98. Solution of a quadratic stability Ulam type problem, Archivum Mathematicum **40**(2004), 1-16 pp (.pdf file).
- 99. *Asymptotic behavior of mixed type functional equations*, Austr. J. Math. Anal. and Appl. **1**(2004), 1-21pp.
- 100. The Ulam stability problem in approximation of approximately quadratic mappings by quadratic mappings ,
   J. Inequ. Pure and Appl.Math.,5(2004),Issue 3,1-9pp.
- 101. On the Heisenberg-Weyl Inequality, J. Inequ. Pure and Appl.Math.,6(2004),Issue ,1-8pp.
- 102. On the Ulam stability for Euler-Lagrange type quadratic functional equations, (with M. J. Rassias),
  Austral. J. Math. Anal. Appl. 2(2005), Issue 1, 1-10pp.
- 103. Asymptotic behavior of Jensen and Jensen type functional equations, (with M. J. Rassias), PanAmerican Math.J. **15**(2005),No4,21-35.
- 104. On the refined Heisenberg-Weyl type inequality,J. Inequ. Pure & Appl. Math. 6(2005), Issue 2, 1-11pp.
- 105. Asymptotic behavior of alternative Jensen and Jensen type functional equations, (with M. J. Rassias),
  - Bulletin Sciences Mathematiques 129(2005), in press, Issue 7, 545-558pp.
- 106. Alternative Contraction Principle and Ulam Stability Problem, Math. Sci. Res J. **9**(7) 2005 190-199pp.
- 107. Solution of the Hyers-Ulam stability problem for quadratic type functional equations in several variables, Austral. J. Math. Anal. –electronic 2(2005), Issue 2, 1-9 pp.
- 108. On the general quadratic functional equation, Bol. Soc. Mat. Mexicana (3)**11**(2005), 259-268 pp.
- 109. On the Ulam problem for Euler quadratic mappings, Novi Sad J. Math. 35(2)(2005), 57-66.
- 110. On the Cauchy-Ulam stability of the Jensen equation in C\*-algebras, Internat. J. Pure & Appl. Math. Sci. 2(1)(2005), 92-101.
  - 111.On the general quadratic functional equation. *Libertas Math.* **25** (2005), 151—160.
  - 112.(with Matina (Stamatiki) J. Rassias) Refined Hyers-Ulam approximation for Jensen and Euler-Lagrange mappings, *dedicated volume, National and Kapodistrian University of Athens*, 2006, 417-437.
  - 113. Alternative contraction principle and alternative Jensen and Jensen type mappings, *Intern. J. Appl. Math. & Stat.* 4(M06)(2006), 1-10.
  - 114.(with Matina J. Rassias) The Ulam problem for 3- dimensional quadratic mappings. *Tatra Mt. Math. Publ.* **34** (2006), , part II, 333-337.
  - 115.Refined Hyers Ulam approximation of approximately Jensen type mappings, *Bulletin Sci. Math.* **131** (2007), no. 1, 89-98.

- 116.(with Hark-Mahn Kim and Young-Sun Cho) Stability problem of Ulam for Euler-Lagrange quadratic mappings. J. Inequal. Appl. 2007, Art. ID 10725, 1-15.
- 117.(with Hark- Mahn Kim and Kil-Woung Jun) Extended stability problem for alternative Cauchy-Jensen mappings. J. Inequal. Pure Appl. Math. 8 (2007), no. 4, Article 120, 1-17.
- 118.(with Hark- Mahn Kim and Kil-Woung Jun) Extended Hyers-Ulam stability for Cauchy-Jensen mappings. *J. Difference Equ. Appl.* **13** (2007), no. 12, 1139-1153.
- 119.(with Hark- Mahn Kim) Generalization of Ulam stability problem for Euler-Lagrange quadratic mappings. *J. Math. Anal. Appl.* **336** (2007), no 1, 277-296.
- 120.(with Xiang, Shuhuang and Matina J. Rassias) On the Aleksandrov and triangle isometry Ulam stability problems. *Intern. J. Appl. Math. Stat.* 7 (2007), No Fe07, 133-142.
- 121.(with Matina J. Rassias) Refined Ulam stability for Euler-Lagrange type mappings in Hilbert spaces. *Intern. J. Appl. Math. Stat.* **7** (2007), No. Fe07, 126-132.
- 122.(with Chun-Gil Park) Hyers-Ulam stability of an Euler-Lagrange type additive mapping. *Int. J. Appl. Math. Stat.* **7** (2007), No. Fe07,112-125.
- 123. (with M. S. Moslehian) Power and Euler-Lagrange Norms, *Austral. J. Math. Anal. Appl.*.,4(2007), 1-4.
- 124.Leonhard Paul Euler, His Life and His Work, F. I. D. A., Intern. J. Appl. Math. Stat., 7(Fe07) (2007), 8-17.
- 125.Tricomi-Protter problem of nD mixed type equations, *Intern. J. Appl. Math. Stat.*, 8(M07) (2007), 76-86.
- 126.(with A. Hasanov) Fundamental solutions of two degenerated elliptic equations and solutions of boundary value problems in infinite area, *Intern. J. Appl. Math. Stat.* 8(M07)(2007), 87-95.
- 127.(with G. C. Wen) Solvability of the oblique derivative problem for second order equations of mixed type with nonsmooth degenerate curve, *Intern. J. Appl. Math. Stat.* 8(N07)(2007), 96-111.
- 128.(with Soon-Mo Jung) Stability of general Newton functional for logarithmic equations spirals. *Adv. Difference Equ.* 2008, Art. 143053, 1-5.
- 129.(with K. Ravi and M. Arunkumar) Ulam stability for the orthogonally general Euler-Lagrange type functional equation, *Intern. J. Appl. Math. Stat.* 3 (2008), A08, 36-46.
- 130.Mixed type partial differential equations with initial and boundary values in fluid mechanics, *Intern. J. Appl. Math. & Stat.*, 13(J08)(2008), 77-107.

### PUBLICATIONS IN SPECIALIZATION AREAS

The following sections contain selected scientific publications of the author according to his specialization areas in Mathematical Analysis

### **1. MIXED TYPE PARTIAL DIFFERENTIAL EQUATIONS**

#### **Books Authored**

- 1. Mixed type Equations and Maximum Principles in fluid dynamics, 1983, Greece
- 2. Applied Analysis : Partial Differential Equations, Vol. II, 2003,
  - Symmetry Publ. , Greece
- 3. Lecture Notes on Mixed Type Partial Differential Equations, World Scientific, 144 pp,1990
- 4. Counter Examples in Differential Equations and Related Topics, World Scientific, 192 pp,1991

#### **Selected Citations**

- W. F. Ames (GIT) for the Mathematical Reviews, AMS: Any analyst is always concerned about good counterexamples to use in lectures and research. Here is a book with a collection of counterexamples in the areas of continuity, differentiability, extrema, existence, uniqueness, stability, regularity, periodicity, etc. They have been collected from a variety of books and journals. The collection should prove useful to analysts interested in the main topic of differential equations. There is a reasonable bibliography and both a subject and an author index. This is a valuable book for students and researchers alike.
- -G.C.Rota (M.I.T.) for the World Scientific : The author knows his business : he has chosen the right counterexamples at the right time. The section on plane autonomous systems is particularly felicitous, and should be consulted by whoever teaches elementary differential equations ( some of the examples are really cute and we will use them in our 400-student course next year ). Why can't mathematicians write more books like this one ?

### **Books Edited**

- Mathematics Space Technology, Greece, 1980
- Mathematical Analysis, Teubner-Texte zur Mathematik, Vol. 79, 1985
- Mixed type Equations, Teubner-Texte zur Mathematik, Vol. 90, 1986
- Mathematical Equations and Inequalities, Vol. I, Greece, 1999
- Mathematical Equations and Inequalities, Vol. II, Greece, 1999

### **Published Papers**

1. Weak solutions of the Frankl-Morawetz problem in  $\mathbb{R}^{n+1}$   $(n \ge 2)$ , (with G. Rassias, Th. Rassias), Tamkang J. Math. **10** (1979), no. 1, 81-91

- 2. On the Frankl problem of second kind, Tamkang J. Math. **10** (1979), no. 2, 231-236
- 3. *The 3-dimensional Frankl problem*, Bull. Soc. Roy. Sci. Liege **48** (1979), no. 11-12, 422-423
- 4. *The Bitsadze-Lavrentjev problem*, Bull. Soc. Roy. Sci. Liege **48** (1979), no. 11-12, 424-425
- 5. *A new mixed type boundary value problem*, Bull. Soc. Roy. Sci. Liege **48** (1979), no. 11-12, 420-421
- 6. *The survey on equations of mixed type*, "Functional differential systems and related topics" (Proc. First Internat. Conf., B \ I a\.zejewko,1979), 295-301, Higher College Engrg., Zielona Gora, (1980)
- 7. A maximum principle in  $\mathbb{R}^3$ , C. R. Math. Rep. Acad. Sci. Canada 2 (1980), no.3, 131-133
- New uniqueness theorems, Bull. Acad. Polon. Sci. Ser. Sci. Math. 28 (1980), no. 11-12, 569-571
- 9. *A new bi-hyperbolic boundary value problem in the Euclidean space*, Bull. Acad. Polon. Sci. Ser. Sci. Math. **28** (1980), no. 11-12, 565-568
- On a defective theorem on elliptic hyperbolic equations, Bull. Soc. Roy. Sci. Liege 49 (1980), no. 9-10, 307-309
- 11. Weak solutions for a mixed type problem, Bull. Soc. Roy. Sci. Liege **49** (1980), no. 5-8, 278-280
- 12. On a Goursat type problem, C. R. Math. Rep. Acad. Sci. Canada 2 (1980/81), no. 1, 49-51
- 13. Uniqueness and existence theorems for a mixed type equation, Tamkang J. Math. **12**, (1981), no. 1, 77-83
- 14. A new mixed type boundary value problem, Bull. Sci. Math. (2) **105** (1981), no. 3, 329-336
- 15. *Mixed type partial differential equations in R<sup>n</sup>*, Tamkang J. Math. **12** (1981), no. 2, 177-181
- A uniqueness theorem for the generalized Frankl-Tricomi problem, Bull. Sci. Math. (2) 105 (1981), no. 3, 321-327
- 17. A maximum principle in R<sup>n+1</sup>, J. Math. Anal. Appl. 85 (1982), no. 1, 106-113
- Weak solutions of the Frankl problem in the 4- dimensional Euclidean space, Bull. Acad. Polon. Sci. Ser. Sci. Math. 30 (1982), no. 3-4, 123-130
- An application of the theory of positive symmetric systems to a degenerate multidimensional hyperbolic equation in R<sup>3</sup>, Serdica 8 (1982), no. 3, 235-242
- 20. An extended Chaplygin problem and a uniqueness theorem for the Chaplygin Frankl problem, Bull. Soc. Roy. Sci. Liege **51** (1982), no. 3-4, 156-160
- 21. *The Bi-hyperbolic Degenerate Boundary Value Problem in R*<sup>3</sup>, Discuss. Math., Vol. 5, (1982), 101-104
- 22. *The extended Bitsadze-Lavrentjev-Tricomi boundary value problem*, Rend. Circ. Mat. Palermo (2) **33** (1984), no.2, 255-264
- 23. On the Tricomi problem with two parabolic lines of degeneracy, Bull. Inst. Math. Acad. Sinica **12** (1984), no. 1, 51-56
- 24. On the exterior mixed type boundary value problem in the Euclidean plane, "Mathematical Analysis", 269-284, Teubner-Texte Math., 79, Teubner, Leipzig, (1985)
- 25. Extended Bitsadze-Lavrentjev problem with two parabolic lines of

*degeneracy and two elliptic arcs in Euclidean plane*, C. R. Acad. Bulgare Sci. **38** (1985), no. 1, 31-34

- 26. *The mixed Bitsadze-Lavrentjev-Tricomi boundary value problem*, "Mixed Type Equations", 6-21, Teubner-Texte Math., 90, Teubner, Leipzig, (1986)
- 27. On three new uniqueness theorems of the Tricomi probem for nonlinear mixed type equations, "Mixed Type Equations", 269-279, Teubner-Texte Math., 90, Teubner, Leipzig, (1986)
- 28. *The mixed Bitsadze-Lavrentjev-Tricomi boundary value problem*, J. Math. Res. Expositions **7** (1987) ,no.1 , 77-80
- 29. On three new generalized uniqueness theorems of the Tricomi problem for nonlinear mixed type equations, J. Math. Phys. Sci. 22 (1988), no. 6, 681-695
- 30. On the well-posedness of the extended Chaplygin problem in a multidimensional region, C. R. Acad. Bulgare Sci. **41** (1988), no. 2, 35-37
- 31. *The well-posed Tricomi-Bitsadze-Lavrentjev problem in the Euclidean plane*, Atti Accad. Sci. Torino Cl. Sci. Fis. Mat. Natur. **124** (1990), no. 3-4, 73-83
- 32. *The exterior Tricomi and Frankl problem*, J. Math. Res. Exposition **10** (1990) no. 4, 485-493
- 33. On the well-posedness of the extended Tricomi-Chaplygin-Frankl problem in a multidimenional region, Chinese. J. Math. **19** (1991), no.3, 187-203
- 34. On the well-posed Tricomi problem in  $\mathbb{R}^2$ , Discuss. Math. 12 (1992), 85-93
- 35. *The well-posed Tricomi problem of two kinds*, J. Math. Phys. Sci. **27** (1993), no. 6, 383-393

World Sci. Publishing, River Edge, NJ, (1994)

- 36. *The well-posed Tricomi problem in the Euclidean plane*, "Geometry, Analysis and Mechanics", 189-195, World Sci. Publishing, River Edge, NJ, (1994)
- 37. Uniqueness of quasi-regular solutions for a parabolic elliptic-hyperbolic Tricomi problem, Bull. Inst. Math. Acad. Sinica **25** (1997), no.4, 277-287
- Bitsadze-Lavrentjev Problem, Encyclopaedia of Mathematics, KluwerAcademic Publishers, file: B: rassi 1, March 28, (1997), 1-4, The Netherlands
- 39. Existence of weak solutions for a parabolic elliptic-hyperbolic Tricomi problem, Tsukuba J. Math. 23 (1999), no. 1, 37-54
- 40. Uniqueness of quasi-regular solutions for a bi-parabolic elliptic bi-hyperbolic Tricomi problem, Complex Var. Theory Appl.(Taylor & Francis) 47 (2002), no. 8, 707-718.
- 41. Mixed type partial differential equations with initial and boundary values in *fluid mechanics*, Intern. J. Appl. Math. & Stat., 13(J08)(2008), 77-107.

### 2. FUNCTIONAL EQUATIONS AND INEQUALITIES

#### **Books Authored**

1. *Mathematical Analysis* : *Multivariable Calculus*, Vol. III, 1996, Symmetry Publ., Greece

#### **Books Edited**

- Functional Analysis, Approximation Theory and Numerical Analysis, World Scientific, 340 pp,1994
- Geometry, Analysis and Mechanics, World Scientific, 388 pp, 1995
- Advances in Equations and Inequalities, Hadronic Press, USA, 1999
- Mathematical Equations and Inequalities, Vol. I, Greece, 1999
- Mathematical Equations and Inequalities, Vol. II, Greece, 1999

### **Published Papers**

- 1. On approximation of approximately linear mappings by linear mappings, J. Funct. Anal. **46** (1982), no. 1, 126-130
- On approximation of approximately linear mappings by linear mappings, Bull. Sci. Math. (2) 108 (1984), no. 4, 445-446
- 3. On a new approximation of approximately linear mappings by linear mappings, Discuss. Math. 7 (1985), 193-196
- 4. Solution of a problem of Ulam, J. Approx. Theory 57 (1989), no. 3, 268-273
- 5. Solution of a stability problem of Ulam, Discuss. Math. 12 (1992), 95-103
- On the stability of the Euler-Lagrange functional equation , Chinese J. Math. 20 (1992) , no. 2 , 185-190
- Stefan Banach, Alexander Markowic Ostrowski, Stanislaw Marcin Ulam, "Functional analysis, approximation theory and numerical analysis", 241-249, World Sci. Publishing, River Edge, NJ, (1994)
- 8. Complete solution of the multi dimensional problem of Ulam, Discuss. Math. **14** (1994), 101-107
- 9. On the stability of the multi-dimensional nonlinear Euler-Lagrange functional equation, "Geometry, Analysis and Mechanics", 275-285, World Sci. Publishing, River Edge, NJ, (1994)
- 10. On the stability of the non-linear Euler-Lagrange functional equation in real normed linear spaces, J. Math. Phys. Sci. **28** (1994), no.5, 231-235
- On the stability of a multi-dimensional Cauchy type functional equation, "Geometry, Analysis and Mechanics", 365-376, World Sci. Publishing, River Edge, NJ, (1994)
- 12. Solution of a stability problem of Ulam, "Functional analysis, approximation theory and numerical analysis", 241-249, World Sci. Publishing, River Edge, NJ, (1994)
- 13. On the stability of the general Euler Lagrange functional equation, Demonstratio Math. 29 (1996), no. 4, 755-766
- 14. Solution of the Ulam stability problem for Euler-Lagrange quadratic mappings, J. Math. Anal. Appl. **220** (1998), no. 2, 613-639
- 15. Solution of the Ulam stability problem for quartic mappings, Glasnik Matematicki Ser. III **34 (54)** (1999), no. 2, 243-252
- 16. On the stability of the multi-dimensional Euler-Lagrange functional equation, J. Indian Math. Soc. (N. S.) 66 (1999), no. 1-4, 1-9
- 17. Solution of the General Dth Degree Functional equation, (A.E.I. : Advances in Equations and Inequalities), Hadronic Press, Fl., USA, (1999), 185-189
- Generalization of the Euler Theorem to Heptagons leading to a Quadratic Vector Identity, (A.E.I. : Advances in Equations and Inequalities), Hadronic Press, Fl., USA, (1999), 179-183
- 19. Augustin Louis Cauchy, His Life and His Work, (A.E.I.: Advances in

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- 20. On the Euler stability problem, J. Indian Math. Soc. (N.S.) **67** (2000), no.1-4, 1-15
- Solution of the Ulam stability problem for nonlinear five-dimensional Euler quadratic mappings, Southeast Asian Bull. Math. (Springer-Verlag) 24 (2000), no. 4, 617-621
- 22. On approximation of approximately quadratic mappings by quadratic mappings, Ann. Math. Sil., No. 15 (2001), 67-78
- 23. Solution of a quadratic stability Hyers-Ulam type problem, Ricerche Mat. 50 (2001), no. 1, 9-17
- 24. Solution of a Cauchy Jensen stability Ulam type problem, Archivum Mathematicum (Brno) **37** (2001), no.3, 161-177
- 25. Solution of the Ulam stability problem for cubic mappings, Glasnik Matematicki Ser. III **36** (**56**) (2001), no.1, 63-72
- 26. *Hyers–Ulam stability for a quadratic functional equation in several variables*, J. Indian Math. Soc. (N. S.) **68** (2001), no.1-4, 65 73
- 27. On the Ulam stability of mixed type mappings on restricted domains,
  J. Math. Anal. Appl. 276 (2002), no. 2, 747-762
- 28. On the Hyers-Ulam stability problem for quadratic multi-dimensional mappings, Aequationes Math. (Birkhauser-Verlag) 64 (2002), no.1-2, 62-69
- 29. Solution of the Ulam stability problem for an Euler type quadratic functional equation, Southeast Asian Bull. Math. **26** (2002), 101-112
- On the Hyers-Ulam stability problem for quadratic multi-dimensional mappings on the Gaussian plane, Southeast Asian Bull. Math. 26 (2002), 483-502
- 31. On some approximately quadratic mappings being exactly quadratic, (with M. J. Rassias), J. Ind. Math. Soc. **69** (2002), 155-160.
- 32. On the Ulam stability of Jensen and Jensen type mappings on restricted domains, (with M. J. Rassias), J. Math. Anal. Appl. **281** (2003),516-524
- 33. On the quadratic functional inequality involving a sum of powers of norms, Intern. J. Math. Sciences 2 (2003), no. 1, 173-184
- 34. *On the general quadratic functional equation*, Libertas Math., 23(2003),165-174.
- 35. Solution of a quadratic stability Ulam type problem, Archivum Mathematicum 40(2004), 1-16 pp. (.pdf file)
- 36. The Ulam stability problem on approximation of approximately quadratic mappings by quadratic mappings,
  - J. Inequ. Pure & Appl. Math. 5(2004), Issue, 1-9pp.
- 37 . *Asymptotic behavior of mixed type functional equations*, Austr. J. Math. Anal. and Appl. 1(2004), 1-21pp.
- 38. Asymptotic behavior of Jensen and Jensen type functional equations, (with M. J. Rassias), PanAmerican Math. J. **15**(2005), No4, 21-35.
- 39. Asymptotic behavior of alternative Jensen and Jensen type functional equations, (with M. J. Rassias),
  - Bulletin Sciences Mathematiques 129(2005), Issue 7, 545-558pp.
- 40. *Alternative Contraction Principle and Ulam Stability Problem,* Math. Sci. Res J. **9**(7) 2005 190-199pp.
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- 42. On the general quadratic functional equation, Bol. Soc. Mat. Mexicana (3)11(2005), 259-268 pp.
- 43. On the Ulam problem for Euler quadratic mappings, Novi Sad J. Math. 35(2)(2005), 57-66.
- 44. On the Cauchy-Ulam stability of the Jensen equation in C\*-algebras, Internat. J. Pure & Appl. Math. Sci. 2(1)(2005), 92-101.
- 45. Alternative contraction principle and alternative Jensen and Jensen type mappings, Internat. J. Appl. Math. & Stat. 4(M06)(2006), 1-10.
- 46. *Refined Hyers Ulam approximation of approximately Jensen type mappings*, Bulletin Sci. Math. (Article in press) (2006), 1-10.
- 47.On the general quadratic functional equation. Libert. Math. 25(2005),151-160.

48. Alternative contraction principle and alternative Jensen and Jensen type mappings, Internat. J. Appl. Math. & Stat. 4(M06)(2006), 1-10.

- 49. (with Matina J. Rassias) The Ulam problem for 3- dimensional quadratic mappings. *Tatra Mt. Math. Publ.* 34 (2006), , part II, 333-337.
- 50. Refined Hyers Ulam approximation of approximately Jensen type mappings, Bulletin Sci. Math. 131 (2007), no. 1, 89-98.
- (with Hark-Mahn Kim and Young-Sun Cho) Stability problem of Ulam for Euler-Lagrange quadratic mappings. J. Inequal. Appl. 2007, Art. ID 10725, 1-15.
- 52. (with Hark- Mahn Kim and Kil-Woung Jun) Extended stability problem for alternative Cauchy-Jensen mappings JIPAM. J. Inequal.. Pure Appl. *Math.* 8 (2007), no. 4, Article 120, 1-17.
- (with Hark- Mahn Kim and Kil-Woung Jun) Extended Hyers-Ulam stability for Cauchy-Jensen mappings. J. Difference Equ. Appl. 13 (2007), no. 12, 1139-1153.
- 54. (with Hark- Mahn Kim) Generalization of Ulam stability problem for Euler-Lagrange quadratic mappings. J. Math. Anal. Appl. 336 (2007), no 1, 277-296.
- 55. (with Xiang, Shuhuang and Matina J. Rassias) On the Aleksandrov and triangle isometry Ulam stability problems. *Int. J. Appl. Math. Stat.* 7 (2007), No Fe07, 133-142.
- 56. (with Matina J. Rassias) Refined Ulam stability for Euler-Lagrange type mappings in Hilbert spaces. *Int. J. Appl. Math. Stat.* 7 (2007), No. Fe07, 126-132.
- 57. (with Chun-Gil Park) Hyers-Ulam stability of an Euler-Lagrange type additive mapping. *Int. J. Appl. Math. Stat.* 7 (2007), No. Fe07,112-125.
- 58. (with Soon-Mo Jung) Stability of general Newton functional for logarithmic equations spirals. *Adv. Difference Equ.* 2008, Art. 143053, 1-5.
- 59. (with K. Ravi and M. Arunkumar) Ulam stability for the orthogonally general Euler-Lagrange type functional equation. *Int. J. Math. Stat.* 3 (2008), A08, 36-46.

### **3. OPERATOR THEORY**

#### **Books Authored**

• Applied Analysis : Partial Differential Equations, Vol. II, 2003, Symmetry Publ., Greece

#### **Books Edited**

- Mathematical Analysis, Teubner-Texte zur Mathematik, Vol. 79, 1985
- Functional Analysis, Approximation Theory and Numerical Analysis, World Scientific, 340 pp,1994
- Geometry, Analysis and Mechanics, World Scientific, 388 pp, 1995
- Mathematical Equations and Inequalities, Vol. I, Greece, 1999
- Mathematical Equations and Inequalities, Vol. II, Greece, 1999

### **Published Papers**

- Some fixed point theorems in nonlinear analysis, (with G. Rassias, Th. Rassias) "Functional differential systems and related topics" (Proc. First Internat. Conf., B \ I a\.zejewko,1979), 302-305, Higher College Engrg., Zielona Gora (1980) 445-446
- 2. On the generalized Cesaro operators, Mathematical Analysis, 32-34, Teubner-Texte Math., 79, Teubner, Leipzig, (1985)
- 3. Landau's type inequalities, C. R. Acad. Bulgare. Sci. 46 (1993), no.10, 9-11
- 4. *Open problems in analysis*, "Geometry, analysis and mechanics", 355-364, World Scientific Publshing Co., Inc., River Edge, NJ, (1994)
- 5. Stefan Banach, Alexander Markowic Ostrowski, Stanislaw Marcin Ulam, "Functional analysis, approximation theory and numerical analysis", 241-249, World Sci. Publishing, River Edge, NJ, (1994)
- 6. New Landau's type inequalities, Discuss. Math. 14 (1994), 77-99
- 7. *Fractional linear algebra*, "Geometry, Analysis and Mechanics", 251-267, World Sci. Publishing, River Edge, NJ, (1994)
- 8. *Multi-dimensional Landau inequalities*, "Geometry, Analysis and Mechanics", 287-354, World Sci. Publishing, River Edge, NJ, (1994)
- 9. On the Heisenberg-Weyl Inequality, J.Inequ.Pure and Appl.Math.,6(2004),Issue 1,1-8pp.
- 10. Landau's type inequalities, "Functional analysis, approximation theory and numerical analysis", 281-301, World Sci. Publishing, River Edge, NJ, (1994)
- 11. *Generalized Landau's type inequalities*, "Functional analysis, approximation theory and numerical analysis", 303-325, World Sci. Publishing, River Edge, NJ, (1994)
- 12. Landau's type inequalities, J. Math. Anal. Appl. **202** (1996), no. 1, 280-301
- Multi-dimensional Landau extremum problems, C. R. Acad. Bulgare Sci. 50 (1997), no. 2, 5-8
- 14. Multi-dimensional Landau extremum problems, J. Indian Math. Soc. (N. S.) 66 (1999), no. 1-4, 11-16
- 15. Six-dimensional Landau inequalities, Demonstratio Math. **32** (1999), no.2, 413-431
- 16. Mathematical Computation of the Code of a Date, (A. E. I. : Advances in

Equations and Inequalities), Hadronic Press, Fl., USA, (1999), 191-210 17. *Augustin Louis Cauchy*, *His Life and His Work*, (A. E. I. : Advances in Equations and Inequalities), Hadronic Press, Fl., USA, (1999), 1-25.

18. Leonhard Paul Euler, His Life and His Work, (F. I. D. A.: Functional Equations, Integral Equations, Differential Equations and Applications), Intern. J. Appl. Math. Stat., 7(Fe07)(2007), 8-17.

# 4. MATHEMATICAL INEQUALITIES

### **Books Authored**

• Probability Theory and Statistics, 1989, Symmetry Publ., Greece

#### **Books Edited**

- Mathematical Equations and Inequalities, Vol. I, Greece, 1999
- Mathematical Equations and Inequalities, Vol. II, Greece, 1999

### **Published Papers**

- 1. *A counter example to a conjecture by P. Erdos*, (with G. Rassias , Th. Rassias) Proc. Japan Acad. Ser. A Math. Sci. **53** (1977) , no. 4 , 119-121.
- Research Problems in Complex Analysis (New Problem : No. 6.81), (with G. Rassias, Th. Rassias), (Eds.: D. A. Bannan and J. G. Clunie, Aspects of Contemp. Complex Analysis), Acad. Press, (1980), 584.
- 3. On the derivative of a polynomial, Bull. Soc. Roy. Sci. Liege **51** (1982), no. 9-10, 379-380.
- 4. Two new criteria on characterizations of inner products, Discuss. Math. 9 (1988), 255-267.
- 5. Four new criteria on characterizations of inner products, Discuss. Math. 10 (1990), 139-146.
- 6. *Archimedes*, "Geometry, Analysis and Mechanics", 1-4, World Sci. Publishing, River Edge, NJ, (1994).
- 7. On the extended Ostrowski constant, "Functional analysis, approximation theory and numerical analysis", 237-239, World Sci. Publishing, River Edge, NJ, (1994).
- 8. On the Heisenberg Pauli Weyl inequality, J. Inequ. Pure & Appl. Math., 5(2004), Issue 1,1-70 pp.
- 9. *On the Heisenberg-Weyl Inequality*, J.Inequ.Pure and Appl.Math.,6(2004),Issue 1,1-8pp.
- 10. On the refined Heisenberg-Weyl type inequality, J.Inequ.Pure & Appl.Math. 6(2005), Issue 2, 1-11pp.
- 11. Power and Euler-Lagrange Norms, Austral. J. Math. Anal. Appl., 4 (2007), 1-20.

### **Selected Research Problems**

• J. Hadamard ("Lectures on Cauchy's Problem in Linear Partial Differential Equations", Silliman Lectures Series, Yale University Publications, 1921) coined the name *Cauchy problem*. The following initial value problem or Cauchy problem is one of the major problems of the theory of partial differential equations.

Initial Value Problem or Cauchy Problem: consists in finding a function u = u(x,t) satisfying the hyperbolic equation  $u_{xx} - u_{tt} = 0$  and the initial or Cauchy data u(x,0) = f(x),  $u_t(x,0) = g(x)$ .

The two names *initial value problem* and *Cauchy problem* are actually synonymous. In general, we consider the Cauchy problem for the partial differential equation :  $u_{xt} = f(x,t,u,u_x,u_t)$  (\*) where the function f on the right need *not* be analytic but must satisfy smoothness requirements in its dependence on the arguments  $x, t, u, p = u_x, q = u_t$ .

General Cauchy Problem : asks for a solution u = u(x,t) of the equation (\*) with the property that prescribed values : u = u(s), p = p(s), q = q(s) of u, p and q are assumed along a given initial curve C : x = x(s), t = t(s).

The data u, p and q must fulfill the *compatibility condition*   $\frac{du}{ds} = p\frac{dx}{ds} + q\frac{dt}{ds}$  along the above initial curve C if the function uis to have p and q as its first partial derivatives. Therefore p and qcannot be assigned independently. It is actually the values of u and of its normal derivative  $\frac{\partial u}{\partial v}$  that can be prescribed as arbitrary functions along C. These quantities are usually named *Cauchy data*. This initial value problem is one of the basic core of problems concerning the classical equations of mathematical physics.

• In 1913, E. Landau (Proc. London Math. Soc. 13 (1913), no.2, 43-49) initiated the following extremum problem for twice differentiable functions.

Landau Extremum Problem : The sharp inequality between the supremum-norms of derivatives of twice differentiable functions f such that :  $||f'||^2 \le 4||f|| ||f''||$  holds with norm referring to the space  $C[0,\infty]$ .

If f is a real-valued function defined on  $R = (-\infty, \infty)$ ,  $||f|| = \sup\{|f(x)|: x \in R\}$  and f is twice differentiable and both f and f'' are bounded, J. Hadamard (Comptes Rendus Acad. Sci. Paris 41 (1914), 68-72) achieved the best possible constant 2 in this case. For  $C(-\infty, \infty)$ , A. N. Kolmogorov (Ucen. Zap. Moskov. Gos. Unive., Mat. 30, (1939), 3-16 ; Amer. Math. Soc. Transl. 4, New York, (1949), 233-243) established the above inequality with the same constant 2 and generalized this inequality to derivatives of order higher than 2. Besides, R. R. Kallman & G. C. Rota ("Inequalities, II" (O. Shisha, Ed.), Academic Press, New York, (1970), 187-192) demonstrated that the constant 4, is true also for a semigroup of linear contractions. Moreover, H. Kraljevic & S. Kurepa (Glas. Mat. 5 (1970), 109-117) established the constant 4/3 for a strongly continuous cosine function of linear contractions with an infinitesimal generator. In addition, Z. Ditzian (Aequat. Math. 12 (1975), 145-151) achieved the constant 2 for a group of linear isometries. For a real-valued function f defined on  $(0, \infty)$ ,

define  $||f|| = \left(\int_{0}^{\infty} f^{2}(x)dx\right)^{\frac{1}{2}}$ . If f is twice differentiable and both f and f'' are bounded, G. H. Hardy ; J. E. Littlewood ; and G. Polya (Proc. London Math. Soc. 25 (1926), no. 2, 265-282 ; "Inequalities", (1934) Cambridge , Univ. Press , England ) showed the above inequality with 2 the best possible constant . Moreover , these three authors (1934) showed that the general inequality  $||f^{(k)}||^{n} \le ||f||^{n-k} ||f^{(n)}||^{k}$ , 0 < k < n holds with 1 the best possible constant , if f is a real-valued function on  $(-\infty,\infty)$  and  $||f|| = \left(\int_{-\infty}^{\infty} f^{2}(x)dx\right)^{\frac{1}{2}}$  as well as f is n-differentiable and both f and  $f^{(n)}$  are bounded. This extremum problem is interesting in operator theory and approximation theory , as well .

• In 1923, F. G. Tricomi (Atti Accad. Naz. Lincei, 14 (1923), 133-247) initiated the work on boundary value problems for partial differential equations of mixed type and related equations of variable type. The Tricomi equation  $yu_{xx} + u_{yy} = 0$  plays a central role in the mathematical analysis of transonic flow. As the simplest equation with that property, it provides a useful mathematical model of the transition from subsonic to supersonic speeds in aerodynamics.

Tricomi Problem or Problem T: consists in finding a function u = u(x, y) which satisfies the Tricomi equation :  $yu_{xx} + u_{yy} = 0$  (\*) in a mixed domain D which is simply connected and bounded by a Jordan (non-selfintersecting) "elliptic" arc  $g_1$  (for y > 0) with endpoints O = (0,0)

and A = (1, 0) and by the "real or hyperbolic" characteristics  $g_2$ :  $x + \frac{2}{3}(-y)^{3/2} = 1$ ,  $g_3$ :  $x - \frac{2}{3}(-y)^{3/2} = 0$  of (\*) (for y < 0) satisfying the characteristic equation:  $y(dy)^2 + (dx)^2 = 0$  such that these characteristics meet at a point P (for y < 0) and u assumes prescribed continuous boundary values  $u = \varphi(s)$  on  $g_1$  and  $u = \psi(x)$  on  $g_3$  (\*\*).

In 1935, S. Gellerstedt (Doctoral Thesis, Uppsala, 1935; Jbuch Fortschritte Math. 61 (1935), 1259) generalized the *problem* T by replacing the coefficient y of  $u_{xx}$  in the above equation (\*) by  $sgn(y)|y|^m$ , m > 0. In 1945, F. I. Frankl (Izv. Akad. Nauk SSSR ser. mat. 9; Bull. de l'Acad. des Sci. de l' URSS, 9 (1945), no.2, 121-143) established a generalization of the problem T for the *Chaplygin equation*:  $K(y)u_{xx} + u_{yy} = 0$  with K(y) > 0 for y > 0; < 0 for y < 0; K(0) = 0. We note that this equation was established in 1904 by S. A. Chaplygin ("On Gas Jets", Scientific Annals of the Imperial University of Moscow, Publication no.21, 1904; translation: Brown Univ., R. I., 1944).

Frankl Problem or Problem F: consists in finding a function u = u(x, y) which satisfies the Chaplygin equation:  $K(y)u_{xx} + u_{yy} = 0$  (\*\*) in a mixed domain D which is simply connected and bounded by a Jordan "elliptic" arc  $g_1$  (for y > 0) with endpoints O = (0, 0) and A = (1, 0), by the real characteristic  $g_2$ :  $x = \int_0^y \sqrt{-K(t)} dt + 1$  of (\*\*) (for y < 0) satisfying the characteristic equation:  $K(y)(dy)^2 + (dx)^2 = 0$  and by the non-characteristic triangle OAP and intersecting the characteristic  $g_2$  at most once  $(g'_3 \text{ may coincide with the "real" characteristic <math>g_3$ :  $x = -\int_0^y \sqrt{-K(t)} dt$  of (\*\*) (for y < 0) near the point O) and assuming prescribed continuous boundary values  $u = \varphi(s)$  on  $g_1$  and  $u = \psi(x)$  on  $g'_3$ .

F. I. Frankl (in 1945) initiated a new stage in the theory of equations of mixed type. In particular, he established the uniqueness of the solution of the above Problem F in the case where *the Frankl condition*: F(y)=1+2(K/K')'>0, for y < 0 holds with derivative K'(y) > 0. Note that this condition is equivalent to the convexity of  $(-K)^{-1/2}$  for y < 0. M. A. Lavrentjev and A. V. Bitsadze

(Dokl. Akad. Nauk. SSSR 70, 3, 1950, 373-376) suggested the wellknown *Bitsadze - Lavrentjev model* with a discontinuous K = sgn(y). According to M. H. Protter (Bull. Amer. Math. Soc., 1 (1979), no. 3, 534-538) the task of forming a single comprehensive theory for mixed type equations in two dimensions appears formidable; the development in three and more dimensions is even more remote. M. H. Protter (J. Rat. Mech. & Anal. 2 (1953), no. 1, 107-114) improved the above Frankl condition. Besides, Protter (J. Rat. Mech. & Anal. 3 (1954), no. 4, 435-446) was the first to consider the case in three dimensions. These boundary value problems are important in fluid dynamics.

• In 1927, W. Heisenberg (Zeit. Physik 43 (1927), 172 - ; Univ. Chicago Press, 1930 ; and Dover edit., New York, 1949 ) demonstrated the impossibility to specify simultaneously the position and the momentum of an electron within an atom. The following result named , Heisenberg uncertainty inequality, is not actually due to Heisenberg. In 1928, according to H. Weyl (S. Hirzel, Leipzig, 1928; and Dover edit., New York, 1950) this result is due to W. Pauli.

*Heisenberg* Uncertainty Inequality: If  $f: R \to C$  is a complex valued function of a random real variable x such that  $f \in L^2(R)$ , then the product of the second moment

of the random real x for  $|f|^2$  and the second moment of the random real  $\xi$  for  $|\hat{f}|^2$ 

is at least  $\int_{R} |f(x)|^2 dx / 4\pi$ , where  $\hat{f}$  is the Fourier transform of f, such that  $\hat{f}(\xi) = \int_{R} e^{-2i\pi\xi x} f(x) dx$  and  $f(x) = \int_{R} e^{2i\pi\xi x} \hat{f}(\xi) d\xi$  with  $i = \sqrt{-1}$ .

According to N. Wiener ("the Fourier integral and certain of its applications", Cambridge, 1933) a pair of transorms cannot both be very small. This inequality plays an important role in different aspects of Fourier and Time – Frequency Analysis. A huge number of well-written books and overview-papers deals with uncertainty relations.

• In 1932, S. Banach ("Theorie des Operations Lineaires", Monografje Matematyczne, Warsaw, 1932, Poland) introduced the normed linear spaces. In 1935, P. Jordan & J. von Neumann (Ann. Math. 36 (1935), 719-723) established the following parallelogram equality characterization of inner product spaces.

Jordan - von Neumann Characterization of Inner Products : A normed linear space V is an inner product space if the parallelogram equality :  $\|v - w\|^2 + \|v + w\|^2 = 2\|v\|^2 + 2\|w\|^2$  ( $v, w \in V$ ) holds.

A Banach space whose norm satisfies this law is a Hilbert space. G. Birkhoff (Duke Math. J., 1 (1935), 169-172), R. C. James (Duke Math. J., 12 (1945), 291-301; Bull. Amer. Math. Soc. 53 (1947), 559-566) and M. M. Day (Trans. Amer. Math. Soc., 62 (1947), 320-337) gave basic characterizations of inner products by orthogonality relations and by the duality map. Moreover, a major improvement was the idea of I. J. Schoenberg (Proc. Amer. Math. Soc., 3 (1952), 961-964) to replace the parallelogram equality by an inequality. The most "natural" geometric properties may fail to hold in a general normed space unless the space is an inner (or:scalar) product space. These characterizations of inner products or Hilbert spaces are important in the geometry of Banach spaces and approximation theory.

• In 1940, S. M. Ulam proposed, before the Mathematics Club of the University of Wisconsin, the following problem on homomorphisms.

Ulam Problem : given a group  $G_1$  and a metric group  $G_2$  with metric d(.,.) as well as  $\varepsilon > 0$ , does there exist a  $\delta > 0$  such that if  $f:G_1 \to G_2$  satisfies  $d(f(xy), f(x)f(y)) < \delta$  for all  $x, y \in G_1$ , then a homomorphism  $h:G_1 \to G_2$  exists with  $d(f(x), h(x)) < \varepsilon$  for all  $x \in G_1$ ?

In 1968, Ulam ("a collection of mathematical problems", Intersci. Publ., Inc., New York, 1968, p. 63) posed the following more general problem.

*General Ulam Problem*: When is it true that by changing slightly the hypotheses of a theorem one can still assert that the thesis of the theorem remains true or approximately true?

D. H. Hyers (Proc. Nat. Acad. Sci., 27 (1941), no. 4, 222-224) considered the case of approximately additive mappings  $f: X \to Y$ satisfying the functional inequality  $||f(x+y)-f(x)-f(y)|| < \varepsilon$  for all x,  $y \in X$ , where X and Y are Banach spaces. Then he showed that  $A(x) = \lim 2^{-n} f(2^n x)$  exists for all  $x \in X$  and that  $A: X \to Y$ 

that  $A(x) = \lim_{n \to \infty} 2^{-n} f(2^n x)$  exists for all  $x \in X$  and that  $A: X \to Y$ is the unique additive mapping satisfying  $||f(x) - A(x)|| \le \varepsilon$  for all  $x \in X$  with  $\varepsilon > 0$ .

According to P. M. Gruber (Trans. Amer. Math. Soc. 245 (1978), 263-277) this Ulam problem is of particular interest in probability theory and in the case of functional equations of different types. There is now an extensive research work on the stability of the Ulam problem for functional equations with applications in probability theory, financial and actuarial mathematics.

# Teaching

### I. Undergraduate program:

- **1. Theory of Probability and Statistics.** Random experiment and Sample space. Classical and conditional probability. Bayes'formulas. Random variable. Stochastic function and Probability distributions. Normal distribution.
- **2.** Mathematics III. Limits, derivatives, simple integrals and applications. Partial differentiation. Extrema subject to constraints. Method of Lagrange Multipliers.
- **3.** Number Theory. Numbers and Divisibility. Diophantine equations. Congruences. Mathematical computation of the Orthodox Easter by Gauss.
- **4. Mathematical Programming.** Gauss-Jordan elimination method. Geometric Method. Simplex Method. Duality principle and dual problem by John von Neumann. Applications
- **5.** Mathematical Topics. Continued fractions. Fibonacci numbers and recursive sequences. Lucas numbers. Golden-section number. Applications. Center of gravity and centroid. Fractions, equations and analysis in Music.

### II. <u>Graduate program:</u>

- **1. Analysis.** Multiple integration. Functional equations and inequalities, Ulam stability problem.
- **2.** Tutorial Analysis , Classical inequalities. Landau differential inequalities and Heisenberg integral inequalities.
- 3. History of Analysis, Famous Problems and Theorems with their History.
- **4. Combinatorial Analysis and Computational Mathematics.** Combinations and Permutations. Applications. Mixed type equations and Tricomi boundary value problem.