Cauchy Problem

John Michael Rassias

- J. Hadamard ("Lectures on Cauchy's Problem in Linear Partial Differential Equations" , Silliman Lectures Series , Yale University Publications , 1921) coined the name *Cauchy problem* . The following initial value problem or Cauchy problem is one of the major problems of the theory of partial differential equations .

Initial Value Problem or Cauchy Problem : consists in finding a function $u = u(x,t)$ satisfying the hyperbolic equation $u_{xx} - u_{tt} = 0$ and the initial or Cauchy data $u(x,0) = f(x)$, $u_t(x,0) = g(x)$.

The two names *initial value problem* and *Cauchy problem* are actually synonymous . In general , we consider the Cauchy problem for the partial differential equation : $u_{xt} = f(x,t,u,u_x,u_t)$ $(*)$ where the function f on the right need *not* be analytic but must satisfy smoothness requirements in its dependence on the arguments x , t , u , $p = u_x$, $q = u_t$.

General Cauchy Problem : asks for a solution $u = u(x,t)$ of the equation $(*)$ with the property that prescribed values : $u = u(s)$, $p = p(s)$, $q = q(s)$ of u , p and q are assumed along a given initial curve $C : x = x(s)$, $t = t(s)$.

The data u , p and q must fulfill the *compatibility condition* $\frac{du}{ds} = p\frac{dx}{ds} + q\frac{dt}{ds}$ along the above initial curve C if the function u is to have p and q as its first partial derivatives . Therefore p and q cannot be assigned independently . It is actually the values of u and of its normal derivative $\partial u / \partial \nu$ that can be prescribed as arbitrary functions along C . These quantities are usually named *Cauchy data* . This initial value problem is one of the basic core of problems concerning the classical equations of mathematical physics .